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LETTER TO THE EDITOR

Crossover scaling of rough surfaces: effects of surface diffusion with threshold

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Abstract. Surface properties of a random deposition model in which the effects of surface diffusion with the threshold Δh are taken into account are studies in $1+1$ dimensions. The surface diffusion occurs only when the difference of the height between the nearest neighbours is more than the threshold Δh. The scaling properties of the surface of the deposit are studied as a function of the threshold Δh of surface diffusion. It is shown that a crossover phenomenon in the surface structure of the deposit occurs from the no diffusion region on small length scales to the surface diffusion region on large length scales. The exponent β, which describes how the surface thickness grows with the height of the deposit (for an extremely large deposit), varies from 1/2 (for high thresholds) to 1/4 (for low thresholds). The crossover length h_c from the random filling deposition to the random deposition with surface diffusion scales as $\Delta h^{1.71}$.

Surface structures of growth processes have recently attracted considerable attention [1]. For the ballistic deposition model [2], the Eden model [3] and the random filling deposition model with surface diffusion [4], it has been shown that the surface of the deposit exhibits a self-affine fractal geometry which can be described in terms of the scaling form

$$\xi = L^{\alpha} f(h/L^{z}) \tag{1}$$

where ξ is the variance in the surface height, h is the mean height of the deposit and L is the lateral size of the deposit. The scaling function $f(x)$ tends to a constant value when x tends to infinity and behaves as x^{β} when x tends to zero. ξ behaves as L^{α} for large h and as h^{β} for large L. In $1+1$ dimensions simulations give $\alpha = 1/2$ and β about 1/3 for the ballistic model [2, 5] and the Eden model [3, 6]. By introducing surface diffusion in the random filling deposition model [4], it has been found that the value of the β exponent becomes 1/4. Also it has been shown that the same value of the β exponent is obtained when surface diffusion or complete restructuring [7] is included in the ballistic deposition model. A theory of deposition processes by Edwards and Wilkinson [8] is able to predict the values of α, β, z for the random filling model with surface diffusion. By solving a linear Langevin equation for the growing surface, they derived the exponents

$$\alpha = (2-d)/2 \qquad \beta = (2-d)/4 \qquad \text{and} \qquad z = 2 \tag{2}$$

in $d+1$ dimensions. In $1+1$ dimensions, the result $\alpha = 1/2$, $\beta = 1/4$ and $z = 2$ is in excellent agreement with simulations [4]. This work was later extended by Karder *et al* [9] who took into account the possibility of lateral growth. They showed that the exponents become $\alpha = 1/2$, $\beta = 1/3$ and $z = 3/2$ in $1+1$ dimensions. It appears that the result ($\beta = 1/4$) for deposition models with restructuring can be accounted for by a theory of Edwards and Wilkinson [8], while the value β in the ballistic model without restructuring ($\beta = 1/3$) can be accounted for by a theory of Kardar *et al* [9].

Very recently, Jullien and Meakin [10] studied the influence of a finite concentration of falling particles in ballistic deposition with restructuring in $1+1$ dimensions. They found that the exponent β for an extremely large deposit varies from 1/4 (for small concentrations) to 1/3 (for large concentrations). Also a roughening phase transition in surface growth was found in both $2+1$ and $3+1$ dimensions [11]. Krug [12] proposed a classification of deposition processes which explains why some of models are governed by the linear theory [8].

In this letter we consider the effects of surface diffusion with threshold on the scaling structure of surface properties in the random filling deposition model. The surface diffusion occurs only when the difference of the height between the nearest neighbours is greater than the threshold Δh. The growth of an interface from a substrate can be described by

$$\frac{\partial h}{\partial t} = D\nabla\cdot(\theta(\nabla h - \Delta h)\nabla h) + \eta(x, t) \tag{3}$$

where $h(x, t)$ is the height of the interface at position x and time t, D is the surface diffusion constant, $\theta(x)$ is the step function, Δh is the threshold, and $\eta(x, t)$ is white noise in space and time. In the limit of $\Delta h \to 0$, equation (3) reduces to the linear equation of Edwards and Wilkinson [8].

Firstly, we explain the random filling model with surface diffusion which was proposed by Family [4]. Particles simply rain down onto a substrate. Particles move along straight line trajectories until they reach the top of the column in which they are dropped. A deposited particle diffuses around on the surface within a prescribed region about the column in which it is dropped until it finds the column with the smallest height. At this point the particle sticks to the top of that column and becomes part of the aggregate. In the absence of this diffusive motion, the process is a random filling process in which there is no correlation between columns. The height of the column in the case with no diffusion follows a Poisson process and correspondingly the surface width ξ diverges with the square root of h, independent of L. With the introduction of surface diffusion the surface becomes smoother. The surface width can be written in the scaling form (1) with $\alpha = 1/2$, $\beta = 1/4$ and $z = 2$. In our model surface diffusion occurs only when the difference of the heights between the nearest neighbours is greater than a critical value Δh. In the limit of $\Delta h \to 0$, the surface width scales as $h^{1/4}$ for an extremely large deposit in $1+1$ dimensions. In the limit of $\Delta h \to \infty$, the surface width scales as $h^{1/2}$. Accordingly, with the introduction of the threshold into surface diffusion, a crossover phenomenon is expected to occur from the no diffusion region on small length scales to the surface diffusion region on large length scales.

We study the random deposition model on a square lattice in which particles are deposited from above onto a line of L sites representing the initial nucleation seeds. We employ periodic boundary conditions so that columns i and $i+L$ are equivalent. A newly arriving particle is allowed to diffuse around nearest neighbours if the difference of the heights between the nearest neighbours is greater than a critical value Δh, i.e. a particle dropped in column i sticks to the top of column i, $i+1$ or $i-1$, depending on the heights of the three columns. Examples of deposits introducing surface diffusion with the threshold are shown in figure 1. With decreasing critical value Δh, the surface of the deposit becomes smoother. The surface width ξ of the surface is plotted against

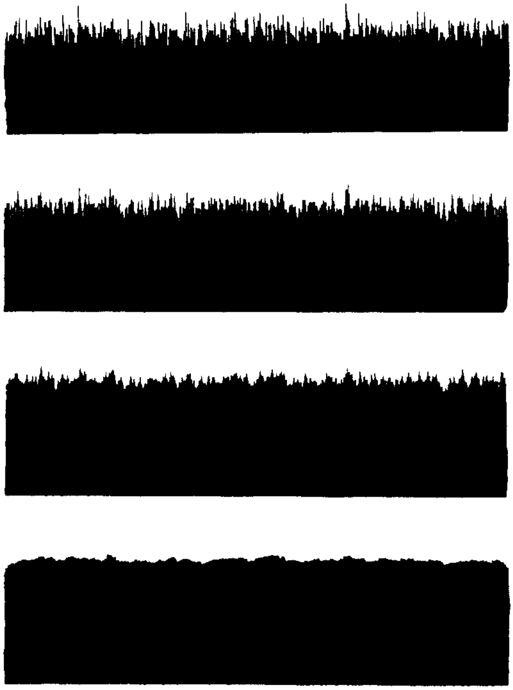

Figure 1. Typical deposits obtained for the threshold $\Delta h = 0$, 5, 10 and ∞. These simulations were performed with $L = 600$ and the maximum height 150. With decreasing Δh, the surface becomes smoother.

the mean height of the deposit (log-log plot) for $\Delta h = 30$, 25, 20, 15, 10, 5, 2.5 and 0. ξ is the result of one simulation with $L = 10\,000$ and h varying up to 3000. The exponent β can be estimated from the curves of figure 2. In the limit of $\Delta h \to \infty$ the slope of the curve is 1/2 and in the limit of $\Delta h \to 0$ the slope is 1/4. The crossover is observed from

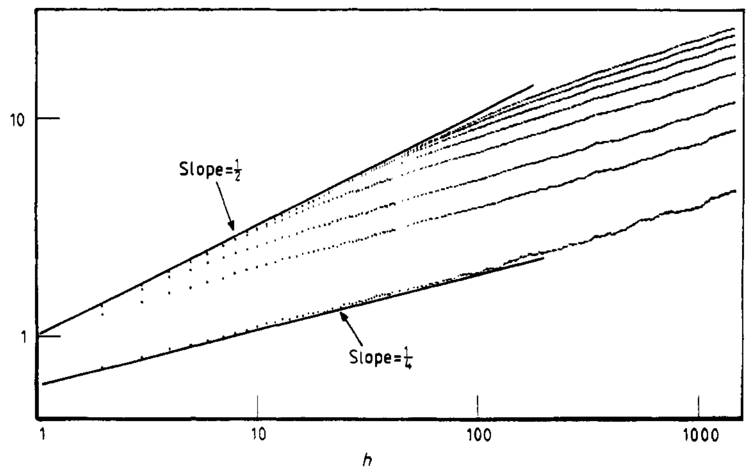

Figure 2. Surface width, ξ, of the surface as a function of the mean height of the deposit, h (log-log plot) for $\Delta h = 0$, 2.5, 5, 10, 15, 20, 25 and 30. The crossover from the slope 1/2 to the slope 1/4 is observed.

no diffusion on small length scales to surface diffusion on large length scales. We plot the crossover length against the threshold Δh in figure 3. We can find the crossover exponent $\phi = 0.58$. We propose the following scaling form for an extremely large deposit:

$$\xi = h^{1/2} g((1/\Delta h) h^{\phi}) \tag{4}$$

with

$$g(x) \approx \begin{cases} \text{constant} & \text{if } x \ll 1 \\ x^{(1/4-1/2)/\phi} & \text{if } x \gg 1. \end{cases}$$

The crossover length h_c scales as follows:

$$h_c \approx (1/\Delta h)^{-1/\phi} \tag{5}$$

where $\phi = 0.58$. The above scaling form holds for $1 \ll h \ll L$. In the region, $h \gg L$, the following scaling form is satisfied:

$$\xi \approx L^{1/2}. \tag{6}$$

The scaling form unified by (1) and (4) is given by

$$\xi = L^{1/2} f(\{h^{1/2} g((1/\Delta h) h^{\phi})\}^4 / L^2). \tag{7}$$

This implies that ξ behaves as $L^{1/2}$ for $h \gg L$, as $h^{1/4}$ for $h_c < h \ll L$, and as $h^{1/2}$ for $1 \ll h < h_c$.

In conclusion, effects of surface diffusion with the threshold on surface properties are studied in 1 + 1 dimensions. For an extremely large deposit, a crossover phenomenon is found to occur from the no diffusion structure on small length scales to the surface diffusion-dominated region on large length scales. The crossover length from the random filling deposition to the random deposition with surface diffusion is calculated.

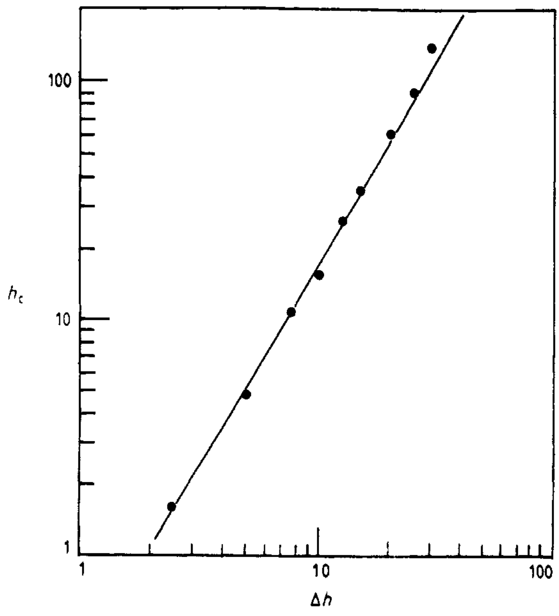

Figure 3. The logarithm of the crossover length h_c is plotted against the logarithm of the threshold Δh. The straight line through the data points indicates that h_c scales as $\Delta h^{1.71}$.

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